

Quantum information transfer for qutrits

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Abstract. We propose a scheme for the transfer of quantum information among distant qutrits. We apply this scheme to the distribution of entanglement among distant nodes and to the generation of multipartite antisymmetric states. We also discuss applications to quantum secret sharing.

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1. Introduction

Transfer and distribution of quantum states play an important role in quantum communications and quantum information processing. Quantum teleportation [1], dense coding [2], and quantum key sharing [3] require the distribution of a maximally entangled state among the involved parties. Distributed quantum computing needs the transfer of arbitrary quantum states among distant nodes. In this context, an ideal scheme for the transfer of quantum states and the distribution of entanglement was proposed in Ref [4, 5]. This scheme considers ions stored in distant cavities as qubits and the transfer of quantum states takes place via a unidirectional exchange of photons triggered by the application of laser pulses on the ions. The extension of this work to the case of perfect transmission in presence of noise channels was also studied [6]. An important experiment in this sense has been recently reported, where the recording of quantum states of light onto an atomic memory with high fidelity has been achieved [7]. Using photons propagating in optical fibres, as flying qubits, and alkaline atoms, as interfaces, transfer of quantum states between photons has been reported [8].

Recently, results from Quantum information theory have been extended to the case of discrete systems of higher dimensions. For instance, it has been shown that cold trapped ions systems, which originally were proposed for implementing a qubit

quantum computer, can be extended naturally to implement a qutrit based quantum computer[13]. Another proposal for quantum computations using qutrits has recently been presented in spin molecule systems [14]. Qutrits have also been proposed to improve the security of quantum key distribution [15].

In this context arises naturally the question about transfer and distribution of higher dimensional quantum states. In this work we address this question. We restrict ourselves to transfer and distribution of qutrits. We consider ions stored in two distant high finesse optical cavities. The transfer of quantum states is achieved through the emission and absorption of polarized photons by cavities containing ions. We apply this scheme to the distribution of entangled states among distant cavities. In particular we study the generation of symmetric and antisymmetric states and connect the results to the problem of quantum secret sharing.

This article is organized as follows: In section 2 we discuss the ideal quantum state distribution protocol. In section 3 we outline the physical model and state the scheme for the transfer of qutrit states. In section 4 we discuss the distribution of entangled states. In section 5 we present a protocol for the generation of symmetric and antisymmetric states. In section 6 we discuss an application of the previous result to the problem of quantum secret sharing. In section 7 we summarize and conclude.

2. Ideal quantum state transfer

In the following we will consider two distinguishable qutrits. Each qutrit is encoded in the electronic levels of a single ion stored in a high finesse cavity. Our aim is to study the conditions to achieve a perfect transfer of an arbitrary quantum state between both qutrits, when the transfer is implemented via the emission of polarized photons from one of the cavities and the subsequent absorption of these photons by the other cavity.

The total system under study consists of both qutrits, the quantized modes of the electromagnetic fields in each cavity and the modes of the electromagnetic field connecting the cavities. The states of the qutrits are given by

$$|\psi\rangle_k = c_0|0\rangle_k + c_{1,l}|1l\rangle_k + c_{1,r}|1r\rangle_k \quad (1)$$

where l and r stand for left and right and $|0\rangle_k$, $|1l\rangle_k$ and $|1r\rangle_k$ are three electronic levels of the first ($k = 1$) or the second ($k = 2$) ion.

The initial state of the total system is given by

$$|\psi_s\rangle = |\psi\rangle_1|0,0\rangle_1|\{0\}\rangle_e|0,0\rangle_2|0\rangle_2, \quad (2)$$

where $|0,0\rangle_l$ denotes the vacuum state of quantized modes at first ($l = 1$) or second ($l = 2$) cavity corresponding to orthogonal polarized modes; $|\{0\}\rangle_e$ represents the vacuum state of the environment modes between cavities.

Under appropriate conditions it is possible to transfer the state of the qutrit to the modes of the first cavity, that is

$$|\psi_s\rangle \rightarrow |0\rangle_1 (c_0|0,0\rangle_1 + c_{1l}|1,0\rangle_1 + c_{1r}|0,1\rangle_1) \otimes |\{0\}\rangle_e|0,0\rangle_2|0\rangle_2. \quad (3)$$

This transformation corresponds to a quantum state swapping between the first ion and the modes of the first cavity. This can be implemented if the electronic transitions $|1l\rangle_1 \rightarrow |0\rangle_1$ and $|2r\rangle_1 \rightarrow |0\rangle_1$ evolve according to an effective Jaynes-Cummings Hamiltonian (with an effective interaction time $\pi = (g\Omega/\Delta)t$, each transition coupled to its corresponding cavity mode.

Now we assume that both polarized photons are emitted from the first cavity to the vacuum modes and subsequently absorbed into the second cavity. Thereafter, the state is

$$|\psi_s\rangle \rightarrow |0\rangle_1|0,0\rangle_1|\{0\}\rangle_e \otimes (c_0|0,0\rangle_2 + c_{1l}|1,0\rangle_2 + c_{1r}|0,1\rangle_2)|0\rangle_2. \quad (4)$$

Thereby, it has been implicitly assumed that the photons are perfectly absorbed at the second cavity.

Finally, the states of the modes in the second cavity are transferred into the electronic levels of the ion, as in the first step of the process. Thus, the final state of the total system is given by

$$|\psi_s\rangle \rightarrow |0\rangle_1|0,0\rangle_1|\{0\}\rangle_e|0,0\rangle_2|\psi\rangle_2, \quad (5)$$

obtaining in this way a state where the information has been perfectly encoded into the second ion.

However, we know that the photons emitted from the first cavity have a nonzero probability of been reflected at the second cavity. Thus, in a complete simulation of the transfer of the qutrit state, we need time dependent Rabi frequencies of classical fields. This allows for Raman processes which are described by effective Jaynes-Cummings interactions.

3. Physical setup and scheme for qutrit's state transfers

Qutrits can be physically realized using the electronic level configuration currently found in experiments of lineal trapped ions. For example, a suitable physical system are ^{138}Ba ions, where the qutrit can be defined in the metastable fine structure corresponding to $4S_{1/2}$ and $5D_{3/2}$ levels. These states can be addressed through Raman transitions such as the one shown in figure 1.

The Hamiltonian operator describing the unitary joint evolution of the cavity modes and the five electronic levels of the ion is

$$\begin{aligned} H_k &= \hbar \sum_{j=l,r} \nu_{j,k} a_{j,k}^\dagger a_{j,k} + \hbar \sum_{i=1}^5 \omega_i |i\rangle_k \langle i|, \\ V_k &= \hbar \sum_{j=l,r} (g_{j,k} a_{j,k} |2j\rangle_k \langle 0| + \Omega_{j,k} |2j\rangle_k \langle 1j| e^{-i\nu_{j,k}t}) + h.c., \end{aligned} \quad (6)$$

where the k index denotes modes and levels at the first ($k = 1$) or second ($k = 2$) cavity. Amplitudes $\Omega_{j,k}(t) = |\Omega_{j,k}(t)|e^{-i\phi_{j,k}(t)}$ represent classical left ($j = l$) and right ($j = r$) polarized pulsed laser fields connecting the transition between levels $|1j\rangle_k$ and $|2j\rangle_k$. The field operators $a_{j,k}$ and $a_{j,k}^\dagger$ describe the transitions between levels $|1j\rangle_k$ and $|0\rangle_k$.

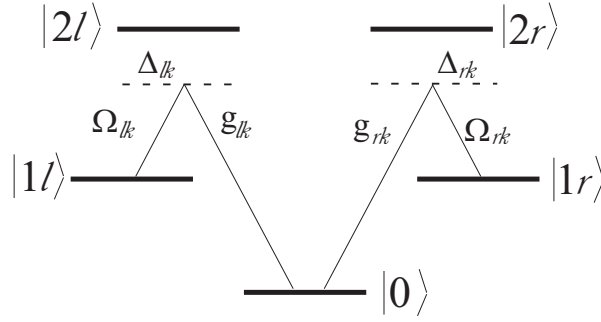


Figure 1. Electronic level structure of trapped ion. Quantum information of qutrits is stored in levels $|0\rangle$, $|1l\rangle$, and $|1r\rangle$.

Assuming that laser and quantum modes are far detuned from the corresponding optical atomic transitions, we can perform an adiabatic elimination of the $|2j\rangle_k$ upper levels, obtaining a pair of effective second order Hamiltonian operators

$$H_{\text{eff}_k} = -\hbar \sum_j \left(\delta_{j,k} a_{j,k}^\dagger a_{j,k} + \frac{|g_{j,k}|^2}{\Delta_{j,k}} a_{j,k}^\dagger a_{j,k} |0\rangle_k \langle 0| + \frac{|\Omega_{j,k}|^2}{\Delta_{j,k}} |j\rangle_k \langle j| + i \frac{\Omega_{j,k}^* g_{j,k}}{\Delta_{j,k}} a_{j,k} |j\rangle_k \langle 0| \right) + h.c., \quad (7)$$

where we have dropped out the label 1 from the electronic levels due to the fact that the $|2j\rangle_k$ states do not appear in the above operator. These Hamiltonian operators allow us to realize the necessary unitary operations to transfer an arbitrary state of the qutrit to the quantum field modes.

Emission from the first cavity and subsequent absorption into the second cavity can be described through cascaded open systems formalism, where the Hamilton operator is given by

$$H_{nh} = -i\hbar \left(\sum_{j,k} \kappa_j \hat{n}_{j,k} + 2\kappa_l a_{l,2}^\dagger a_{l,1} + 2\kappa_r a_{r,2}^\dagger a_{r,1} \right), \quad (8)$$

with κ_j the decay rates of left (l) and right (r) polarized modes of cavities C_1 and C_2 . The global dynamics, including the information transfer into the first cavity, photon emission from the first cavity, absorption in the second cavity, and the final encoding in the second cavity can be described by the Hamiltonian operator

$$H_I = H_{\text{eff}_1} + H_{\text{eff}_2} + H_{nh}. \quad (9)$$

Following reference [?], we consider that no reflected photons from the second cavity are to be detected. Therefore, the following condition must hold:

$$(a_{j,1} + a_{j,2})|\Psi\rangle = 0 \quad j = l, r. \quad (10)$$

In order to solve this problem, we consider only the Hilbert space corresponding to the ions and the quantum fields during the evolution. It is not difficult to realize that

the vector state of the global system can be written as

$$\begin{aligned}
|\psi_s\rangle = & c_0 a(t) |0\rangle_1 |0\rangle_2 |0, 0; 0, 0\rangle \\
& + c_l [(b_{l,1}(t) |l\rangle_1 |0\rangle_2 + b_{l,2}(t) |0\rangle_1 |l\rangle_2) |0, 0; 0, 0\rangle \\
& + |0\rangle_1 |0\rangle_2 (d_{l,1}(t) |1, 0; 0, 0\rangle + d_{l,2}(t) |0, 0; 1, 0\rangle)] \\
& + c_r [(b_{r,1}(t) |r\rangle_1 |0\rangle_2 + b_{r,2}(t) |0\rangle_1 |r\rangle_2) |0, 0; 0, 0\rangle \\
& + |0\rangle_1 |0\rangle_2 (d_{r,1}(t) |0, 1; 0, 0\rangle + d_{r,2}(t) |0, 0; 0, 1\rangle)],
\end{aligned} \tag{11}$$

where we have assumed the amplitudes defined such that $b_{j,k}(t) = \alpha_{j,k}(t)e^{-i\phi_{j,k}(t)}$ with $j = l, r$ and $k = 1, 2$. In order to eliminate the dynamical Stark shifts, we have assumed that the following conditions are satisfied

$$\delta_{j,k} = \frac{|g_{j,k}|^2}{\Delta_{j,k}}, \quad \dot{\phi}_{j,k}(t) = \frac{|\Omega_{j,k}|^2}{\Delta_{j,k}} \tag{12}$$

The state (11), together with the Hamilton operator (9), leads us to the following system of coupled equations for the amplitudes entering into Eq. (11):

$$\begin{aligned}
\dot{a}(t) &= 0, \\
\dot{\alpha}_{j,k}(t) &= -\frac{\Omega_{j,k} g_{j,k}}{\Delta_{j,k}} d_{j,k}(t), \\
\dot{d}_{j,k}(t) &= +\frac{\Omega_{j,k} g_{j,k}}{\Delta_{j,k}} \alpha_{j,k}(t) - \kappa_j (-1)^{k+1} d_{j,k}(t).
\end{aligned} \tag{13}$$

The above equations have been simplified under the no detector click condition for each polarization which ensures that $d_{j,1}(t) + d_{j,2}(t) = 0$. Thus, we have obtained two uncoupled sets of equations, being each set associated with a particular polarization. Each set turns out to be equivalent to the equations obtained in Ref. [?] for the case of ideal transmission of qubit states. To illustrate this let us write the previous set of equations (13) with the definitions $d_{j,1} = (d_{j,s} - d_{j,a})/\sqrt{2}$, $d_{j,2} = (d_{j,s} + d_{j,a})/\sqrt{2}$ and the effective coupling constant $\lambda_{l,k} = \Omega_{j,k} g_{j,k}/\Delta_{j,k}$. In this case we obtain

$$\begin{aligned}
\dot{\alpha}_{j,k}(t) &= (-1)^{k+1} \lambda_{j,k} d_{j,a}(t)/\sqrt{2}, \\
\dot{d}_{j,a}(t) &= -\lambda_{j,1} \alpha_{j,1}(t)/\sqrt{2} + \lambda_{j,2} \alpha_{j,2}(t)/\sqrt{2}.
\end{aligned} \tag{14}$$

In order to achieve an ideal transfer process we impose the following conditions

$$\begin{aligned}
\alpha_{j,1}(-\infty) &= \alpha_{j,2}(\infty) = 1 \quad j = l, r \\
\phi_{j,1}(-\infty) &= \phi_{j,2}(\infty) = 0 \quad j = l, r.
\end{aligned} \tag{15}$$

Additionally, we must take into account the normalization condition. Since the information transfer take place through independent quantum channels, we assume independent normalization conditions for the amplitudes associated with each channel, that is

$$|\alpha_{j,1}|^2 + |\alpha_{j,2}|^2 + |d_{j,a}|^2 = 1 \quad j = l, r. \tag{16}$$

The above system of equations can be solved by imposing the symmetric pulse condition [?] which, in the present context, is given by $\lambda_{j,2}(t) = \lambda_{j,1}(-t)$. These

two conditions together with the set of equations (14) imply $\alpha_{j,1}(t) = \alpha_{j,2}(-t)$ and $d_{j,a}(t) = d_{j,a}(-t)$. Considering the former relations and the evolution equation $\dot{d}_{j,s} = 0$, we obtain

$$\lambda_{j,1}(-t) = -\frac{\sqrt{2}\kappa_j d_{j,a}(t) + \lambda_{j,1}(t)\alpha_{j,1}(t)}{\alpha_{j,2}(t)} \quad t > 0. \quad (17)$$

This expression allows us to determine the pulse shape by means of the calculations in the first half of the time interval. This solution can be in principle addressed once the initial conditions are established. From the normalization condition, symmetry considerations, and the equations $\dot{d}_{j,s}(t) = 0$, the following expression is obtained:

$$2\alpha_{j,1}^2(0)\frac{\lambda_{j,1}(0)^2 + \kappa_j^2}{\kappa_j^2} = 1 \quad j = l, r. \quad (18)$$

Thus, the solution to the problem of transferring the state of a qutrit follows from the transfer of the state of a qubit [?]. This is a consequence of the information channelling through independent dynamics of transitions $|g_k\rangle_a \rightarrow |e_{k1}\rangle_a$ and $|g\rangle_a \rightarrow |e_{k2}\rangle_a$ at each cavity; which turns out to be a consequence of the particular choice for the interaction between the ions and the laser pulses.

4. Distribution of entangled states of qutrits among distant nodes

The above scheme to transfer the state of a qutrit between distant cavities can be applied to the distribution of entanglement among distant qutrits. For sake of simplicity, we shall use the following notation for the ion states $|1l\rangle \rightarrow |1\rangle$ and $|1r\rangle \rightarrow |2\rangle$.

Let us consider for example the following state generated at cavity C_1 between two ions A and B :

$$|\Psi\rangle = \frac{1}{\sqrt{3}}(|0\rangle_a|2\rangle_b + |1\rangle_a|0\rangle_b + |2\rangle_a|1\rangle_b). \quad (19)$$

A third ion C in its ground state is stored in a second distant cavity C_2 . If we apply the scheme for the transfer of qutrit states between ions B and C , we generate the state

$$|\Psi\rangle_{12} = \frac{1}{\sqrt{3}}(|0\rangle_a|2\rangle_c + |1\rangle_a|0\rangle_c + |2\rangle_a|1\rangle_c). \quad (20)$$

Thereby, the entangled state between qutrits A and B has been transferred to the ions B and C .

We can also envisage the distribution of the entangled state of three qutrits from one cavity C_1 to distant cavities C_2 and C_3 . This process is carried out step by step distributing one state at a time. In a physical implementation of this process we should consider a two sided cavity C_1 in order to distribute the states through each side of this cavity to cavities C_2 and C_3 . Thus, it is necessary to include the possibility of controlling the transmission of the photons through the mirrors in C_1 .

5. Generation of an Aharanov-Bohm state using qutrits

In the previous section we discussed the possibility of distributing multiparticle entangled states among distant nodes. In this section we study some special cases.

Let us start by studying the generation of symmetric and antisymmetric states under permutations of individual particles. The generations of such states can be implemented through unitary local transformations and conditional operations. Here we choose the Fourier transform $F^{(D)}$ and the control-not gate $\text{XOR}^{(D)}$ for D -dimensional quantum systems [16].

The conditional operation can be defined as $\text{XOR}_{lmd}^{(D)} |i\rangle |j\rangle = |i\rangle |i \ominus j\rangle$ where $i \ominus j$ is the left modular difference in D dimensions. The operation defined in this way has the property that the inverse operation is the same operation, that is, $(\text{XOR}_{lmd}^{(D)})^2 = I$. This is not true for an alternative conditional operation defined through the modular addition $\text{XOR}_{ma}^{(D)} |i\rangle |j\rangle = |i\rangle |i \oplus j\rangle$, where the inverse is achieved as $(\text{XOR}_{ma}^{(D)})^D = I$. An additional conditional operation that can be defined for D -dimensional systems is the right modular difference $\text{XOR}_{rmd}^{(D)} |i\rangle |j\rangle = |i\rangle |j \ominus i\rangle$; in this case we have $(\text{XOR}_{rmd}^{(D)})^D = I$. These conditional operations are related through local unitary operations.

In D^2 -dimensional spaces the completely symmetric state can be defined generalizing the idea of constructing the two particle state given by:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|i\rangle_1 |j\rangle_2 + |j\rangle_1 |i\rangle_2). \quad (21)$$

There are two states which we can generate in two dimensions, namely, the Bell states $|\Phi_+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$ and $|\Psi_+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$. In both cases we must apply the operations $\text{XOR}_{12}^{(2)} F_1^{(2)} |i\rangle_1 |j\rangle_2$ to the initial root states $|00\rangle$ and $|01\rangle$. This idea is easily extended to D dimensional spaces as follows:

$$|\Psi_{a_1 a_2 \dots a_N}\rangle = U^{(D)} |a_1\rangle \prod_{j=2}^N |a_j\rangle \quad (22)$$

$$U^{(D)} = \prod_{k=2}^N \text{XOR}_{1k}^{(D)} F_1^{(D)}. \quad (23)$$

In the case of two qubits we obtain completely symmetric states under permutation of two particles. However, for D -dimensional systems, the $U^{(D)}$ operation leads to states whose invariance under permutations depends the initial states of the N particles. In the case of qutrits the discrete Fourier transform is defined as:

$$F_n^{(3)} |j\rangle_n = \frac{1}{\sqrt{3}} \sum_{l=0}^2 e^{2i\pi l j / 3} |l\rangle_n. \quad (24)$$

The transformed states $|\bar{j}\rangle = F_n^{(3)} |j\rangle$ read as

$$|\bar{0}\rangle = \frac{1}{\sqrt{3}} (|0\rangle + |1\rangle + |2\rangle),$$

$$\begin{aligned}
|\bar{1}\rangle &= \frac{1}{\sqrt{3}} (|0\rangle + e^{2i\pi/3} |1\rangle + e^{-2i\pi/3} |2\rangle), \\
|\bar{2}\rangle &= \frac{1}{\sqrt{3}} (|0\rangle + e^{-2i\pi/3} |1\rangle + e^{2i\pi/3} |2\rangle).
\end{aligned} \tag{25}$$

Let us consider the conditional $\text{XOR}_{lmd}^D |i\rangle |j\rangle = |i\rangle |i \ominus j\rangle$ operation. Applying the $U^{(D)}$ transformation to initial registers $|0\rangle |1\rangle |2\rangle$ and $|0\rangle |2\rangle |1\rangle$ we obtain the states:

$$\begin{aligned}
|\Psi_{012}\rangle &= \frac{1}{\sqrt{3}} (|0\rangle |2\rangle |1\rangle + |1\rangle |0\rangle |2\rangle + |2\rangle |1\rangle |0\rangle) \\
|\Psi_{021}\rangle &= \frac{1}{\sqrt{3}} (|0\rangle |1\rangle |2\rangle + |1\rangle |2\rangle |0\rangle + |2\rangle |0\rangle |1\rangle).
\end{aligned} \tag{26}$$

These states are invariant under cyclic permutations of the three particle states of each individual system ($021 \rightarrow 102 \rightarrow 210$ and $012 \rightarrow 120 \rightarrow 201$). They are not invariant under the exchange of any two particle states. More general states invariant under the exchange of any two individual particles can be achieved by constructing superpositions of the above states. This can be accomplished by generating a Bell state starting from either the state $|1\rangle |2\rangle$ or the state $|2\rangle |1\rangle$. In the first case we generate the symmetric Bell state between particles 2 and 3 so that

$$\begin{aligned}
|S\rangle_{012} &= \text{XOR}_{12}^{(3)} \text{XOR}_{13}^{(3)} F_n^{(3)} \text{XOR}_{23}^{(2)} H_2^{(2)} |0\rangle |1\rangle |2\rangle \\
|S\rangle_{012} &= \frac{1}{\sqrt{6}} (|0\rangle |1\rangle |2\rangle + |1\rangle |2\rangle |0\rangle + |2\rangle |0\rangle |1\rangle \\
&\quad + |0\rangle |2\rangle |1\rangle + |1\rangle |0\rangle |2\rangle + |2\rangle |1\rangle |0\rangle).
\end{aligned} \tag{27}$$

Thereby, the resulting state is a superposition of $|\Psi_{012}\rangle$ and $|\Psi_{021}\rangle$, leading to the completely symmetric state of three particles.

In the second case we generate the antisymmetric Bell state between particles 2 and 3 so that

$$\begin{aligned}
|A\rangle_{021} &= \text{XOR}_{12}^{(3)} \text{XOR}_{13}^{(3)} F_n^{(3)} \text{XOR}_{23}^{(2)} H_2^{(2)} |0\rangle |2\rangle |1\rangle \\
|A\rangle_{021} &= \frac{1}{\sqrt{6}} (|0\rangle |1\rangle |2\rangle + |1\rangle |2\rangle |0\rangle + |2\rangle |0\rangle |1\rangle \\
&\quad - |0\rangle |2\rangle |1\rangle - |1\rangle |0\rangle |2\rangle - |2\rangle |1\rangle |0\rangle).
\end{aligned} \tag{28}$$

Thus we obtain a superposition of $|\Psi_{012}\rangle$ and $|\Psi_{021}\rangle$, leading to a completely antisymmetric state of three qutrits.

6. Quantum state sharing with qutrits

An interesting application of the states discussed in the previous section arises in the context of quantum state sharing. The main goal is the splitting of a quantum state among several parties such that a subset of the parties can recover the original state only if all the parties agree to cooperate. The state to be shared can be considered as the key to active some process and the scheme attempts to control the misuse of the key by potentially dishonest parties. A quantum state sharing protocol can be formulated using the state $|\Psi\rangle_{021}$, whose generation has been studied in the above section.

The state to be share is

$$|\chi\rangle_3 = c_0 |0\rangle_3 + c_1 |1\rangle_3 + c_2 |2\rangle_3. \quad (29)$$

The joint state of the four qutrits is $|\chi\rangle_3 |\Psi\rangle_{021}$. Qutrit zero and three belongs to the first party, qutrit two belongs to the second party and qutrit three to the third party. Each party corresponds to a cavity with the qutrits physically implemented as trapped ions. The joint initial state obeys the following identity:

$$\begin{aligned} F_1^{(3)} |\chi\rangle_3 |\Psi\rangle_{021} &= \frac{1}{3\sqrt{3}} \sum_{m,\mu} |\Psi_{m,\mu}\rangle_{30} \sum_l Z_1^{1-\mu} |l\rangle_1 \\ &\otimes X_2^{2-\mu} Z_2^{m+l} |\chi\rangle_2. \end{aligned} \quad (30)$$

The states $|\Psi_{m,\mu}\rangle_{03}$ denote the states of qutrits zero and three defined by

$$|\Phi_{m,\mu}\rangle_{30} = \text{XOR}_{30}^3 F_3^3 |m\rangle_3 |\mu\rangle_0. \quad (31)$$

These states form a set of mutually orthogonal, maximally entangled states and can be considered as the generalization of the Bell basis to two qutrits. The operators X and Z are defined as

$$X = \sum_{n=0}^2 |n+1\rangle\langle n|, \quad Z = \sum_{n=0}^2 \omega(n) |n\rangle\langle n|, \quad (32)$$

where $\omega(n)$ are roots of the unity, i.e, $\omega(n) = \exp\left(\frac{2\pi i}{D}n\right)$.

The transference of the quantum state of qutrit zero to qutrit two can be read from Eq. (30). A generalized local Bell measurement on qutrits zero and three projects qutrits two and one to the state

$$\sum_l Z_1^{1-\mu} |l\rangle_1 X_2^{2-\mu} Z_2^{m+l} |\chi\rangle_2. \quad (33)$$

Applying a Fourier transform to qutrit one and measuring it, the state $|\chi\rangle$ is transferred to qutrit three modulo a unitary transformation, that is

$$X_2^{2-\mu} Z_2^{m+l} |\chi\rangle_2, \quad (34)$$

which depends on the outcome (m, μ) of the Bell measurement and on the result (l) of a measurement on qutrit one. In order the third party can recover the state $|\chi\rangle$ on his qutrit, the other parties must agree to share the measurement results.

7. Summary

In this work we have studied the problem of information transfer between quantum systems embodying information in three-dimensional Hilbert spaces. As we have seen, there is an ideal protocol for information transfer having a possible experimental system which could be used to implement it. This physical realization is an extension to qutrit systems of the information transfer proposal between qubits embedded in high finesse optical cavities. As we have shown in a qutrit system, the physical implementation works as if we were transferring information between two independent qubit channels.

In addition, we have discussed the possibility of generating multipartite states among these three-dimensional quantum systems and we have discussed the possibility of using them for quantum state sharing.

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